

# On the possibility of $\eta$ -mesic nucleus formation<sup>\*</sup>

S. A. Sofianos and S. A. Rakityansky

*Physics Department, University of South Africa, P.O.Box 392, Pretoria 0001, South Africa*

Although the  $\eta$ -meson was discovered 40 years ago, only recently particle and nuclear physicists focused their attention on it. In many respects the  $\eta$ -meson is similar to the  $\pi^0$ -meson despite it being four times heavier. Both are neutral, spinless, and have almost the same lifetime,  $\sim 10^{-18}$  sec. The kinship between the two mesons manifests itself very clearly in their decay modes. They are the only mesons which have a high probability of pure radiative decay, i.e., their quarks can annihilate into on-shell photons. The pion almost entirely decays into the radiative channel  $\pi^0 \rightarrow \gamma + \gamma$  (98.798 %). For the  $\eta$  the purely radiative decay is also the most probable mode [1],

$$\eta \rightarrow \begin{cases} \gamma + \gamma & (38.8\%) \\ \pi^0 + \pi^0 + \pi^0 & (31.9\%) \\ \pi^+ + \pi^- + \pi^0 & (23.6\%) \\ \pi^+ + \pi^- + \gamma & (4.9\%) \\ \text{other decays} & (0.8\%) \end{cases} .$$

Therefore, when  $\pi^0$  and  $\eta$  are viewed as elementary particles, they look quite similar. However when one considers their interaction with nucleons, their difference is clearly manifested. Firstly, one expects a manifestation of the large  $\eta\pi^0$ -mass difference in the meson-nucleon dynamics and, at low energies, this is indeed observed. For example, the  $S_{11}$ -resonance  $N^*(1535)$  is formed in both  $\pi N$  and  $\eta N$  systems, but at different collision energies,

$$E_{\pi N}^{res}(S_{11}) = 1535 \text{ MeV} - m_N - m_\pi \approx 458 \text{ MeV}$$

$$E_{\eta N}^{res}(S_{11}) = 1535 \text{ MeV} - m_N - m_\eta \approx 49 \text{ MeV} .$$

Note that due to the large mass of the  $\eta$ -meson (547.45 MeV), this resonance is very close to the  $\eta N$ -threshold. Furthermore it is very broad, with  $\Gamma \approx 150$  MeV, covering the whole low energy region of the  $\eta N$  interaction. As a result the interaction of nucleons with  $\eta$ -mesons in this region, where the  $S$ -wave interaction dominates, is much stronger than with pions. Another consequence of the  $S_{11}$  dominance is that the interaction of the  $\eta$ -meson with a nucleon can be considered as a series of formations and decays of this resonance as shown in Fig. 1.

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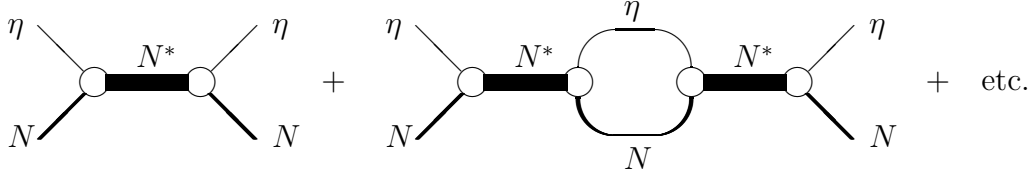


Fig. 1

As with any resonant state, the  $N^*(1535)$ -resonance has branching ratios of the decay modes which do not depend on the formation channel and after its creation it decays into  $\eta N$  and  $\pi N$  channels with equally high probabilities [1]

$$N^*(1535) \rightarrow \begin{cases} N + \eta & (35 - 55 \%) \\ N + \pi & (35 - 55 \%) \\ \text{other decays} & (\leq 10 \%) \end{cases} . \quad (1)$$

Therefore, the series depicted in Fig. 1, must also include terms describing real and virtual transitions into the  $\pi N$ -channel (see Fig 2).

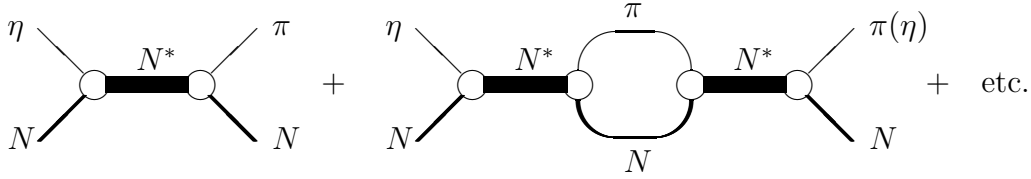


Fig. 2

Thus in the energy region covered by the  $S_{11}$ -resonance, the  $\eta N$  and  $\pi N$  interactions should be treated as a coupled channel problem. When such an analysis was performed, it was found that the near-threshold  $\eta N$  interaction is attractive [2]. This raises the question as to whether this attraction is strong enough so that an  $\eta$ -mesic nucleus can be formed.

Since  $\eta$ -mesons decay very fast, it is impossible to produce beams of them and therefore they can only be observed in final states of certain nuclear reactions with other particles. This makes investigations of  $\eta$ -meson dynamics quite complicated. Therefore the possibility of sustaining an  $\eta$ -meson inside a nucleus would be an exciting one as it would expose itself for a relatively long period in a series of successive interactions with nucleons, i.e., inside the nucleus it would undergo a series of absorptions and emissions through formations and decays of the  $N^*(1535)$ -resonance as depicted in Fig. 3.

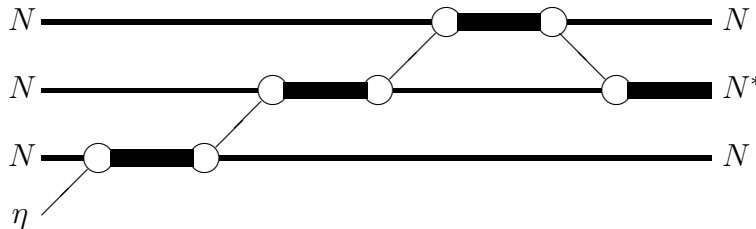


Fig. 3

The lifetime of such an  $\eta$ -mesic nucleus would not be limited by the lifetime of the  $\eta$ -meson itself because after each creation of the  $S_{11}$ -resonance the  $\eta$ -meson is generated anew. However, such an  $\eta$ -nucleus state can not be stable, since eventually the  $N^*(1535)$ -resonance will produce a pion instead of  $\eta$  as their creation probabilities are, according to (1), equally high. Of course such a pion can generate an  $N^*(1535)$ -resonance again which in turn may revive the  $\eta$  but such a possibility is rather low since the pion acquires, through the decay of the resonance, a kinetic energy of  $\sim 400$  MeV and can thus easily escape. It is therefore clear that if an  $\eta$ -meson is bound inside a nucleus, it can only be in a quasi-bound state with nonzero width.

First estimation, obtained in the framework of the optical potential theory, put a lower bound on the number of nucleons  $A$  which is necessary to bind the  $\eta$ -meson, namely,  $A \geq 12$  [3]. Thereafter other theoretical investigations were devoted to this problem. All of them predicted  $\eta$ -nucleus bound states obeying this constraint. However, the search for narrow  $\eta$ -nuclear bound states in an experiment with lithium, carbon, oxygen, and aluminum by Chrien et al. [4] produced negative results.

The conclusion of this experimental work, however, did not discourage theoreticians in examining the possibility of an  $\eta$ -nucleus formation. The relatively large scattering lengths obtained for  $\eta^3\text{He}$  and  $\eta^4\text{He}$  systems using a zero-range  $\eta N$ -interaction [5] cast doubt on the  $A \geq 12$  constraint. Speculations of this kind are based on the argument that in the vicinity of the origin of the complex momentum plane the amplitude  $f$  can be replaced by the scattering length  $a$  and therefore the  $S$ -matrix in this area can be written as

$$S = 1 + 2ikf \approx 1 + 2ika \approx \frac{1 + ika}{1 - ika}.$$

This expression is valid only for small  $k$  and can have a pole in this region only if  $a$  is large. If  $a$  is negative the pole would be on the positive imaginary axis (bound state). Thus, a large negative scattering length indicates that a weakly bound state exists. Decreasing the interaction strength transforms the bound state into a resonance, and vice versa, implying a change of the scattering length from  $-\infty$  to  $+\infty$ .

Such simple reasoning, however, is valid only when the interaction is described by a real potential. In the case of an  $\eta$ -nucleus system the inelastic  $\eta A \rightarrow \pi A$  channel is always open giving rise to a significant imaginary part in the  $\eta$ -nucleus potential. The resonance and quasi-bound state poles of the  $S$ -matrix generated by a complex potential have quite different distribution in the complex  $k$ -plane. In Ref. [6] it was shown that starting from a purely real potential and introducing an imaginary part which is gradually increased, results in  $S$ -matrix pole-behaviour shown in Fig. 4.

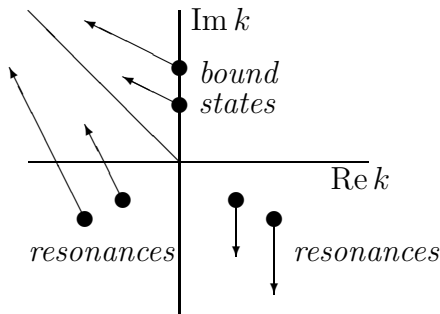


Fig. 4

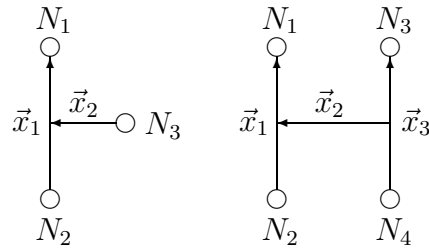


Fig. 5

Therefore, in the case of a complex potential, both resonance and quasi-bound state poles are situated in the second quadrant of the complex momentum plane, under and above its diagonal respectively. The diagonal separates them because the energy  $E_0 = k_0^2/2\mu$  corresponding to a pole at  $k = k_0$ ,

$$E_0 = \frac{1}{2\mu} \left[ (\text{Re } k_0)^2 - (\text{Im } k_0)^2 + 2i(\text{Re } k_0)(\text{Im } k_0) \right] ,$$

has a positive (negative) real part when  $k_0$  is under (above) it. Thus, the transition from resonances to quasi-bound states is a crossing of the diagonal. Since this can take place rather far from the point  $k = 0$ , we should not expect, in contrast to the real potential case, to have a large scattering length even if the binding energy,  $|\text{Re } E_0|$ , is small. Moreover, crossing the diagonal is not associated with dramatic changes of  $a$ . In short, scattering length calculations cannot provide a definite answer and a more rigorous approach must be employed. The most adequate way to solve this problem is to locate the poles of the  $S$ -matrix in the second quadrant of the  $k$ -plane. In Refs. [7,8] we developed a microscopic method that enabled us to calculate the elastic scattering amplitude for any complex value of  $k$  and thereby to locate its poles. The influence of inelastic channels is taken into account via a complex  $\eta N$  potential. In what follows this method is described in somewhat more detail.

Consider the scattering of an  $\eta$ -meson from a nucleus consisting of  $A$  nucleons. The Hamiltonian of the system is given by

$$H = H_0 + V_{\eta A} + H_A \quad (2)$$

where  $H_0$  is the free Hamiltonian corresponding to the  $\eta$ -nucleus relative motion,  $V_{\eta A} = V_1 + V_2 + \dots + V_A$  is the sum of  $\eta N$  potentials,  $V_i \equiv V_{\eta N}(|\vec{R} - \vec{r}_i|)$ , where  $\vec{R}$  and  $\vec{r}_i$  are the coordinates of the  $\eta$  and the  $i$ -th nucleon with respect to the *c.m.* of the nucleus, and  $H_A$  is the total Hamiltonian of the nucleus,

$$H_A = \frac{\hbar}{i} \sum_{i=1}^A \nabla_{\vec{r}_i} + \sum_{i \neq j} V_{NN}(|\vec{r}_i - \vec{r}_j|) . \quad (3)$$

The elastic scattering amplitude  $f(\vec{k}', \vec{k}; z)$  describing the transition from the initial,  $|\vec{k}, \psi_0\rangle$ , to the final,  $|\vec{k}', \psi_0\rangle$ , asymptotic state where  $|\psi_0\rangle$  is the nuclear ground state and  $\vec{k}$  the  $\eta$ -nucleus relative momentum, can be expressed in terms of the T-matrix elements

$$f(\vec{k}', \vec{k}; z) = -\frac{\mu}{2\pi} \langle \vec{k}', \psi_0 | T(z) | \vec{k}, \psi_0 \rangle . \quad (4)$$

The operator  $T$  is related to the Green function  $G_A(z) = (z - H_0 - H_A)^{-1}$  by

$$T(z) = V + V G_A(z) T(z) . \quad (5)$$

The task of solving Eq. (5) is a formidable one and thus one must resort to approximations. One such approximation is the so-called Finite-Rank Approximation (FRA) of the Hamiltonian. It has been proposed in Refs. [9,10] as an alternative to the multiple scattering and optical potential theories. In this method the auxiliary operator

$$T^0(z) = V + VG_0(z)T^0(z), \quad (6)$$

where  $G_0(z) = (z - H_0)^{-1}$  is the free Green function, is introduced. Using the identity  $A^{-1} - B^{-1} = B^{-1}(B - A)A^{-1}$  with  $A = z - H_0 - H_A$  and  $B = z - H_0$ , one gets the resolvent equation

$$G_A(z) = G_0(z) + G_0(z)H_AG_A(z), \quad (7)$$

and thus

$$T(z) = T^0(z) + T^0(z)G_0(z)H_AG_A(z)T(z). \quad (8)$$

The latter equation has the advantage that the spectral decomposition for the Hamiltonian,

$$H_A = \sum_n \mathcal{E}_n |\psi_n\rangle\langle\psi_n| + \int dE E |\psi_E\rangle\langle\psi_E|, \quad (9)$$

can be employed to bring Eq. (5) into a manageable form. The FRA method is based on the approximation

$$H_A \approx \mathcal{E}_0 |\psi_0\rangle\langle\psi_0|, \quad (10)$$

which means that during the scattering of the  $\eta$ -meson, the nucleus remains in its ground state  $|\psi_0\rangle$ . Such an approximation is widely used in multiple scattering and optical potential theories where is known as the coherent approximation. Using (10) we get

$$T(z) = T^0(z) + \mathcal{E}_0 T^0(z) |\psi_0\rangle\langle\psi_0| G_0(z) G_0(z - \mathcal{E}_0) \langle\psi_0| T(z). \quad (11)$$

The matrix elements  $T(\vec{k}', \vec{k}; z) \equiv \langle\vec{k}', \psi_0| T(z) |\vec{k}, \psi_0\rangle$  are thus given by

$$\begin{aligned} T(\vec{k}', \vec{k}; z) &= \langle\vec{k}', \psi_0| T^0(z) |\vec{k}, \psi_0\rangle \\ &+ \mathcal{E}_0 \int \frac{d\vec{k}''}{(2\pi)^3} \frac{\langle\vec{k}', \psi_0| T^0(z) |\vec{k}'', \psi_0\rangle}{(z - k''^2/2\mu)(z - \mathcal{E}_0 - k''^2/2\mu)} T(\vec{k}'', \vec{k}; z). \end{aligned} \quad (12)$$

The auxiliary operator  $T^0$  describes the scattering of the  $\eta$ -meson from nucleons fixed in their space position within the nucleus. This is clear since Eq. (6) does not contain any operator acting on the internal nuclear Jacobi coordinates denoted by  $\{\vec{r}\} \equiv \{\vec{x}_1, \vec{x}_2, \dots, \vec{x}_{A-1}\}$ . Therefore all operators in Eq. (6) are diagonal in the configuration subspace  $\{\vec{r}\}$  and thus

$$T^0(\vec{k}', \vec{k}; \vec{r}; z) = V(\vec{k}', \vec{k}; \vec{r}) + \int \frac{d\vec{k}''}{(2\pi)^3} \frac{V(\vec{k}', \vec{k}''; \vec{r})}{z - k''^2/2\mu} T^0(\vec{k}'', \vec{k}; \vec{r}; z) \quad (13)$$

where

$$\langle\vec{k}', \vec{r}'| T^0(z) |\vec{k}, \vec{r}\rangle = \delta(\vec{r}' - \vec{r}) T^0(\vec{k}', \vec{k}; \vec{r}; z), \quad \langle\vec{k}', \vec{r}'| V |\vec{k}, \vec{r}\rangle = \delta(\vec{r}' - \vec{r}) V(\vec{k}', \vec{k}; \vec{r}).$$

It is clear that  $T^0(\vec{k}', \vec{k}; \vec{r}; z)$  depends parametrically on  $\{\vec{r}\}$ . Therefore the matrix elements  $\langle\vec{k}', \psi_0| T^0(z) |\vec{k}, \psi_0\rangle$  can be obtained by integrating over the Jacobi coordinates

$$\langle \vec{k}', \psi_0 | T^0(z) | \vec{k}, \psi_0 \rangle = \int d\vec{r} |\psi_0(\vec{r})|^2 T^0(\vec{k}', \vec{k}; \vec{r}; z). \quad (14)$$

Thus the solution of the scattering problem can be obtained by solving first Eq. (13), averaging as in Eq. (14), and finally calculating  $T$  from Eq. (12). We must emphasize that the above scheme, is not the same as the first order optical potential approach used in the traditional pion-nucleus multiple scattering theory [11]. Indeed, the latter is based on three approximations; i) the Impulse Approximation; ii) the omission of higher order rescattering terms in constructing the optical potential, and iii) the coherent approximation. In contrast, in the scheme considered here, the Impulse Approximation to obtain the  $\eta N$  amplitude in nuclear media is not needed and no rescattering terms are omitted.

The parameter  $z$  in the above equations corresponds to the total  $\eta$ -nucleus energy,  $z = E - |\mathcal{E}_0| + i0$ , where  $E$  is the energy associated with the  $\eta$ -nucleus relative-motion. On the energy shell we have  $E = k^2/2\mu$ . Therefore, even the auxiliary  $T^0$ -matrix differs from the conventional fixed-scatterer amplitude in that it is always taken off the energy shell. In the case of scattering length calculations, we have  $E = 0$  and thus  $z = -|\mathcal{E}_0|$ . This makes Eqs. (12) and (13) nonsingular and easy to handle.

For practical calculations we rewrite Eq. (13) using the Faddeev-type decomposition

$$T^0(\vec{k}', \vec{k}; \vec{r}; z) = \sum_{i=1}^A T_i^0(\vec{k}', \vec{k}; \vec{r}; z),$$

$$T_i^0(\vec{k}', \vec{k}; \vec{r}; z) = t_i(\vec{k}', \vec{k}; \vec{r}; z) + \int \frac{d\vec{k}''}{(2\pi)^3} \frac{t_i(\vec{k}', \vec{k}''; \vec{r}; z)}{z - k''^2/2\mu} \sum_{j \neq i} T_j^0(\vec{k}'', \vec{k}; \vec{r}; z), \quad (15)$$

where  $t_i$  is the t-matrix for the  $\eta$ -meson scattered by the nucleon  $i$  and is expressed in terms of the two-body  $t_{\eta N}$ -matrix via

$$t_i(\vec{k}', \vec{k}; \vec{r}; z) = t_{\eta N}(\vec{k}', \vec{k}; z) \exp [i(\vec{k} - \vec{k}') \cdot \vec{r}_i]. \quad (16)$$

Expanding the  $|\vec{k}, \vec{r}\rangle$  basis in partial waves and using the fact that, at the low energies considered here, the  $\eta N$  interactions is dominated by the  $S_{11}$ -resonance, we may retain the S-wave only. The total orbital momentum is zero and since the  $\eta$ -meson is a spinless particle the nuclear spin can be ignored. Therefore, when Eq. (15) is projected on the S-wave basis  $|k, r\rangle$ , it reduces to

$$T_i^0(k', k; r; z) = t_i(k', k; r; z) + \frac{1}{2\pi^2} \int_0^\infty dk'' \frac{k''^2 t_i(k', k''; r; z)}{z - k''^2/2\mu} \sum_{j \neq i} T_j^0(k'', k; r; z) \quad (17)$$

where

$$\langle k', r' | T_i^0(z) | k, r \rangle = \frac{\delta(r' - r)}{4\pi r^2} T_i^0(k', k; r; z)$$

and similarly for  $\langle k', r' | t_i(z) | k, r \rangle$ .

The above formulae are given for the general case of  $A$  nucleons. In what follows we restrict ourselves to  $A = 2, 3$ , and 4. The relevant Jacobi vectors are shown in Fig. 5. According to Eq. (16),  $t_i$  depends on the space configuration of the nucleons because  $k$  and  $k'$  are the  $\eta$ -meson momenta with respect to the nuclear centre of mass while the nucleon  $i$

is shifted from it by the vector  $\vec{r}_i = a_i \vec{x}_1 + b_i \vec{x}_2 + c_i \vec{x}_3$ , where  $a_1 = \frac{1}{2}, a_2 = -\frac{1}{2}, b_1 = b_2 = c_1 = c_2 = 0$  for the deuteron case;  $a_1 = \frac{1}{2}, a_2 = -\frac{1}{2}, a_3 = 0, b_1 = b_2 = \frac{1}{3}, b_3 = -\frac{2}{3}, c_1 = c_2 = c_3 = 0$  for the three-nucleon case, and  $a_1 = \frac{1}{2}, a_2 = -\frac{1}{2}, a_3 = a_4 = 0, b_1 = b_2 = \frac{1}{2}, b_3 = b_4 = -\frac{1}{2}, c_1 = c_2 = 0, c_3 = \frac{1}{2}, c_4 = -\frac{1}{2}$  for the four-nucleon case.

The  $S$ -wave projection of Eq. (16) gives

$$\begin{aligned} \langle k', r' | t_i(z) | k, r \rangle &= \int \frac{d\vec{k}'_i d\vec{k}_i}{(2\pi)^6} d\vec{r}'' d\vec{r}''' \langle k', r' | \vec{k}'_i, \vec{r}'' \rangle \langle \vec{k}'_i, \vec{r}'' | t_i(z) | \vec{k}_i, \vec{r}''' \rangle \langle \vec{k}_i, \vec{r}''' | k, r \rangle \\ &= \frac{\delta(r' - r)}{4\pi r^2} j_0(a_i k' x_1) j_0(b_i k' x_2) j_0(c_i k' x_3) t_{\eta N}(k', k; z) j_0(a_i k x_1) j_0(b_i k x_2) j_0(c_i k x_3), \end{aligned}$$

where  $j_0$  is the spherical Bessel function. The  $\eta N$  interaction can be described by the  $t$ -matrix

$$t_{\eta N}(k', k; z) = \frac{\lambda}{(k'^2 + \alpha^2)(z - E_0 + i\Gamma/2)(k^2 + \alpha^2)}. \quad (18)$$

This ansatz is motivated by the  $S_{11}$ -resonance dominance. The vertex function for  $\eta N \leftrightarrow N^*$  is chosen as  $1/(k^2 + \alpha^2)$  which in configuration space has a Yukawa-type behaviour. The propagator is taken to be of a simple Breit-Wigner form. With such a choice the  $t_i$  has the following separable form

$$t_i(k', k, r; z) = H_i(k'; r) \tau(z) H_i(k, r) \quad (19)$$

where

$$\tau(z) = \frac{\lambda}{z - E_0 + i\Gamma/2}, \quad H_i(k, r) = \frac{j_0(a_i k x_1) j_0(b_i k x_2) j_0(c_i k x_3)}{(k^2 + \alpha^2)}. \quad (20)$$

Therefore,

$$T^0(k', k, r; z) = \sum_{i,j=1}^A H_i(k'; r) \Lambda_{ij}(z) H_j(k, r) \quad (21)$$

where

$$(\Lambda^{-1})_{i,j} = \frac{\delta_{ij}}{\tau(z)} - (1 - \delta_{ij}) \Gamma_{ij}(r, z), \quad \Gamma_{ij}(r, z) = \frac{1}{2\pi^2} \int_0^\infty dk \frac{k^2}{z - k^2/2\mu} H_i(k; r) H_j(k, r). \quad (22)$$

Formally, the the last integral involves products of six Bessel functions. However, several of the coefficients  $a_i$ ,  $b_i$ , and  $c_i$  are always zero and therefore only products of at most four Bessel functions can appear in the expression for  $\Gamma_{ij}(r, z)$  with the required integrals having the general form

$$\gamma(p, u, v, w) = \int_0^\infty dk \frac{k^2 j_0(ku) j_0(kv) [j_0(kw)]^2}{(k^2 + \alpha^2)^2 (k^2 - p^2 - i0)}. \quad (23)$$

To calculate the latter integral we introduce the auxiliary one

$$\hat{\gamma}(p, u, v, w, \delta) = \int_0^\infty dk \frac{k^2 j_0(ku) j_0(kv) [\sin(kw)]^2}{(k^2 + \alpha^2)^2 (k^2 - p^2 - i0) (k^2 + \delta^2) w^2} \quad (24)$$

and thereafter evaluate the limit

$$\gamma(p, u, v, w) = \lim_{\delta \rightarrow 0} \hat{\gamma}(p, u, v, w, \delta). \quad (25)$$

The result thus obtained is

$$\gamma(p, u, v, w) = \frac{1}{16uvw^2} [g(u + v + 2w) - 2g(u + v) + g(u + v - 2w) - g(u - v + 2w) + 2g(u - v) - g(u - v - 2w)], \quad (26)$$

where

$$g(s) = \frac{i\pi}{(p^2 + \alpha^2)^2} \left\{ \text{sign}(\text{Im } p) \frac{1}{p^3} \exp[ip|s| \text{sign}(\text{Im } p)] - \frac{i \exp(-\alpha|s|)}{2\alpha^5} [2\alpha^2 + (3 + \alpha|s|)(p^2 + \alpha^2)] - \frac{i|s|(p^2 + \alpha^2)^2}{\alpha^4 p^2} \right\} \quad (27)$$

with

$$\text{sign}(\alpha) = \begin{cases} +1, & \text{for } \alpha \geq 0 \\ -1, & \text{for } \alpha < 0. \end{cases} \quad (28)$$

To obtain the necessary nuclear wave functions  $\psi_0$  we employed the Malfliet–Tjon  $NN$ -potential [12] and the integro–differential equation approach (IDEA) [13,14] which, for  $S$ -wave projected potentials, is equivalent to the exact Faddeev equations.

Using the above formalism, the position and movement of poles of the  $\eta$ -meson–light nuclei ( $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$ , and  $^4\text{He}$ ) elastic scattering amplitude in the complex  $k$ -plane are studied. The two-body t-matrix is assumed to be of the form (18) with  $E_0 = 1535 \text{ MeV} - (m_N + m_\eta)$  and  $\Gamma = 150 \text{ MeV}$  [1]. The range parameter used,  $\alpha = 2.357 \text{ fm}^{-1}$ , was obtained in a two-channel fit to the  $\pi N \rightarrow \pi N$  and  $\pi N \rightarrow \eta N$  experimental data [2]. The parameter  $\lambda$  is chosen to provide the correct zero-energy on-shell limit, i.e., to reproduce the  $\eta N$  scattering length  $a_{\eta N}$ ,

$$t_{\eta N}(0, 0, 0) = -\frac{2\pi}{\mu_{\eta N}} a_{\eta N}.$$

The scattering length  $a_{\eta N}$ , however, is not accurately known. Different analyses [15] provided values for the real part in the range  $\text{Re } a_{\eta N} \in [0.27, 0.98] \text{ fm}$  and for the imaginary part  $\text{Im } a_{\eta N} \in [0.19, 0.37] \text{ fm}$ . Therefore in a search for bound states one must use values of  $a_{\eta N}$  within these ranges. To achieve this we used  $a_{\eta N} = (\zeta 0.55 + i0.30) \text{ fm}$  and vary  $\zeta$  until a bound state appears.

The poles found with  $a_{\eta N} = (0.55 + i0.30) \text{ fm}$  are shown in Fig. 6. The corresponding energies and widths are given in Table 1. When  $\text{Re } a_{\eta N}$  increases, all the poles move up and to the right, and when a resonance pole crosses the diagonal it becomes a quasi-bound pole. The minimal values of  $\text{Re } a_{\eta N}$  which generate ‘zero-binding’ (the poles just on the diagonal) are given in Table 2.

All these values are within the uncertainty interval  $\text{Re } a_{\eta N} \in [0.27, 0.98] \text{ fm}$ . Thus even the possibility of an  $\eta$ d binding cannot be at present excluded. Most recent estimates of  $\text{Re } a_{\eta N}$  [16] are concentrated around the value  $\text{Re } a_{\eta N} \approx 0.7 \text{ fm}$ , which enhances our belief that at least the  $\alpha$ -particle can entrap an  $\eta$ -meson.



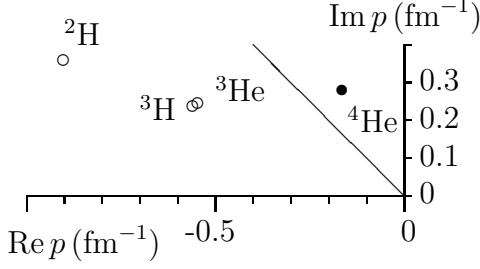


Fig. 6

system	$E$ (MeV)	$\Gamma$ (MeV)
$\eta$ $^2\text{H}$	31.46	59.38
$\eta$ $^3\text{H}$	10.91	22.68
$\eta$ $^3\text{He}$	10.14	22.70
$\eta$ $^4\text{He}$	-2.05	7.48

Table 1

For each of the four nuclei considered, the scattering lengths were calculated with eight values of the strength parameter  $\lambda$  corresponding to  $\text{Re } a_{\eta N}$ :  $\{(0.2 + 0.1n) \text{ fm}; n = 1, 8\}$ , which extends over the uncertainty interval. The  $\text{Im } a_{\eta N}$  was fixed to the value 0.3 fm. An increase of  $\text{Re } a_{\eta N}$  moves the points along the trajectories, shown Figs. 7 and 8, anti-clockwise. When  $\text{Re } a_{\eta N}$  exceeds the critical values given in Table 2, the  $\eta N$  interaction becomes strong enough to generate a quasi-bound state. The corresponding  $\eta$ -nucleus scattering lengths are shown by filled circles (the trajectories for  $^3\text{He}$  and  $^3\text{H}$  are practically the same).

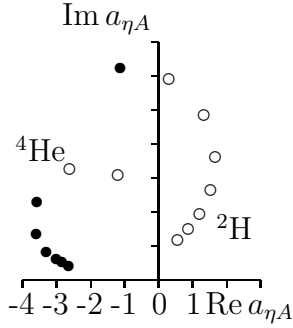


Fig. 7

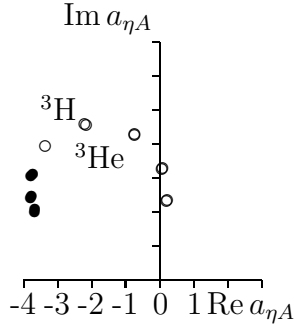


Fig. 8

system	$\min\{\text{Re } a_{\eta N}\}$ (fm)
$\eta$ $^2\text{H}$	0.91
$\eta$ $^3\text{H}$	0.75
$\eta$ $^3\text{He}$	0.73
$\eta$ $^4\text{He}$	0.47

Table 2

Finally, we would like to emphasize that the spectral properties of Hermitian and non-Hermitian Hamiltonians are quite different. Locating quasi-bound states is a delicate problem which can be treated only by rigorous methods. As we have shown in Ref. [8] the  $\eta A$  scattering length tells us nothing about the existence or not of an  $\eta A$  quasi-bound state. This is clearly seen on Figs. 7 and 8 where the trajectories go smoothly from open to filled circles without any drastic changes or extreme values.

In summary, it is shown that within the existing uncertainties of the elementary  $\eta N$  interaction all light nuclei considered can support a quasi-bound state which can result in an  $\eta$ -mesic nucleus which is analogous to hypernuclei. Due to the specific quantum numbers of the  $\eta$ -meson ( $I=0$ ,  $S=0$ ) such states, if they do exist, can be used to access new nuclear states inaccessible by other mesons such as pions and kaons. Furthermore it can be used to elucidate the role played by the  $\eta$  meson in Charge Symmetry Breaking reactions and in the violation of the Okubo-Zweig-Iizuka (OZI) rule.

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